Pre-STEP School online lecture series: Lecture 4 – Question 2

STEP Mathematics III Summer 2006

1 Sketch the curve with cartesian equation

$$y = \frac{2x(x^2 - 5)}{x^2 - 4}$$

and give the equations of the asymptotes and of the tangent to the curve at the origin.

Hence determine the number of real roots of the following equations:

(i) $3x(x^2 - 5) = (x^2 - 4)(x + 3)$;

(ii) $4x(x^2 - 5) = (x^2 - 4)(5x - 2)$;

(iii) $4x^2(x^2 - 5)^2 = (x^2 - 4)^2(x^2 + 1)$.

Please provide answers to the following discussion questions. Don't include full calculations in your responses, just explore the question and try to anticipate routes through it.

1. Applying the standard series of steps to sketch the curve, is there anything that will require special consideration?

2. How do the equation in parts (i), (ii), (iii) relate to the curve you have sketched? How will this help? Will you need to sketch any other curves/lines? If so what will they be? Answer these questions separately for each of (i), (ii), (iii).

Submit your answers by e-mail to stepeasterschool@maths.org by Friday 23rd March 2012 with the subject line: Lecture 4 Question 2